Physics 230

Summary of Results from Lecture — Week of September 8

Pressure is force per unit area, $d\vec{F} = -pd\vec{A}$, where the direction of $\vec{A}$ is outward along the surface normal.

The compressibility of a fluid $k = -(1/V)\partial V/\partial p$. $k = 1/p$ for an ideal gas, but is much smaller for ordinary liquids, which can be regarded as approximately incompressible ($k \approx 0$).

Pressure variation in a column of incompressible liquid $p(z) = p_0 + \rho g(h - z)$ where h is the height of the column. The gradient of the pressure gives the *force per unit volume* $\vec{f} = -\nabla p$. The volume integral of $\int_{\mathcal{V}} \vec{f}\, dV$ is the hydrostatic force on the material in the enclosed volume.

Pascal's principle: An external pressure is transmitted undiminished to all parts of a fluid in equilibrium. Note that this is a statement about the pressure, not the force.

Archimedes principle : The bouyant force on an object in a fluid is the weight of the excluded fluid.

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Force on curved surface: The force from a fluid with (constant) pressure p on a hemisphere of radius R is $\vec{F} = \oint_{\mathcal{S}} p d\vec{A} = \pi p R^2$. In general, if the pressure is constant, this force depends only on the cross sectional area perpendicular to the line of action of the force.

Divergence Theorem: For any vector field $\vec{u}(\vec{r})$, the flux of $\vec{u}$ through a closed surface $\mathcal{S}$ is $\oint_{\mathcal{S}} \vec{u} \cdot d\vec{A} = \int_{\mathcal{V}} \vec{\nabla} \cdot \vec{u}\, dV$. This means the total flow of $\vec{u}$ through the closed surface is the sum of the net flows from each point (aka the *divergence* of $\vec{u}$) contained in the volume bounded by the surface. (Worked example: a calculation of the Archimedes force from the pressure of a fluid at the boundary of an immersed object).

Surface Tension: The surface energy of a fluid is $U_s = \gamma A$ where A is the surface area and γ is the surface tension. $[\gamma] = \mathrm{J/m^2} = \mathrm{N/m}$. The latter is useful for formulating the surface tension force using the contact "line".

Equation of Continuity: For a steady flow bounded by a container of varying cross sectional area A, the volume flow rate $\Phi = vA$ is constant.

Equation of Motion: The force density $\vec{f} = \rho\partial\vec{v}/\partial t + \rho(\vec{v} \cdot \vec{\nabla})\vec{v}$, where the second term is called the "convective derivative." For steady flow we have $\vec{f} = \rho(\vec{v} \cdot \vec{\nabla})\vec{v}$. In these expressions $\vec{v}$ is the average velocity of all the particles found in a fixed volume element of the fluid.

Note: for steady flow in a circular channel $(\vec{v}\cdot\vec{\nabla})\vec{v} = -(v^2/r)\hat{r}$ in each volume element. This gives the net "centripetal acceleration" of the particles in the volume element. Since the flow is steady $\partial\vec{v}/\partial t = 0$.

Bernoulli's Law: For steady, incompressible and irrotational flow, $(1/2)\rho v^2 + \rho g z + p =$ constant. This is the work energy theorem for steady flow in a fluid.

Torricelli's Law: The flow velocity from a liquid column of height h is the free fall velocity $v = \sqrt{2gh}$.

The viscous force transmitted across a surface $\mathcal{A}$ of a flowing liquid is $F/\mathcal{A} = \eta\dot{\varepsilon}$ where $\dot{\varepsilon}$ is the strain rate and η is the viscosity.

Poiseuille's Law: The flow velocity of a viscous fluid in a circular pipe with radius R and length ℓ is *nonuniform*

$$v(r) = \frac{\Delta p}{4\eta\ell}\left(R^2 - r^2)\right) \tag{1}$$

and the volume flow rate is proportional to R^4

$$\Phi = \frac{\pi R^4 \Delta p}{8\eta\ell} \tag{2}$$

Stokes' Law: A sphere of radius R moving at velocity v relative to a fluid with viscosity η experiences a viscous drag force $F_v = 6\pi\eta R v$.