

Pressure is force per unit area,  $d\vec{F} = -pd\vec{A}$ , where the direction of  $\vec{A}$  is outward along the surface normal.

The compressibility of a fluid  $k = -(1/V)\partial V/\partial p$ .  $k = 1/p$  for an ideal gas, but is much smaller for ordinary liquids, which can be regarded as approximately incompressible ( $k \approx 0$ ).

Pressure variation in a column of incompressible liquid  $p(z) = p_0 + \rho g(h - z)$  where  $h$  is the height of the column. The gradient of the pressure gives the *force per unit volume*  $\vec{f} = -\nabla p$ . The volume integral of  $\int_V \vec{f} dV$  is the hydrostatic force on the material in the enclosed volume.

*Pascal's principle*: An external pressure is transmitted undiminished to all parts of a fluid in equilibrium. Note that this is a statement about the pressure, not the force.

*Archimedes principle*: The bouyant force on an object in a fluid is the weight of the excluded fluid.

*Force on curved surface*: The force from a fluid with (constant) pressure  $p$  on a hemisphere of radius  $R$  is  $\vec{F} = \oint_S pd\vec{A} = \pi pR^2$ . In general, if the pressure is constant, this force depends only on the cross sectional area perpendicular to the line of action of the force.

*Divergence Theorem*: For any vector field  $\vec{u}(\vec{r})$ , the flux of  $\vec{u}$  through a closed surface  $\mathcal{S}$  is  $\oint_S \vec{u} \cdot d\vec{A} = \int_V \vec{\nabla} \cdot \vec{u} dV$ . This means the total flow of  $\vec{u}$  through the closed surface is the sum of the net flows from each point (aka the *divergence* of  $\vec{u}$ ) contained in the volume bounded by the surface. (Worked example: a calculation of the Archimedes force from the pressure of a fluid at the boundary of an immersed object).

*Surface Tension*: The surface energy of a fluid is  $U_s = \gamma A$  where  $A$  is the surface area and  $\gamma$  is the surface tension.  $[\gamma] = \text{J/m}^2 = \text{N/m}$ . The latter is useful for formulating the surface tension force using the contact "line".

*Equation of Continuity*: For a steady flow bounded by a container of varying cross sectional area  $A$ , the volume flow rate  $\Phi = vA$  is constant.

*Equation of Motion*: The force density  $\vec{f} = \rho \partial \vec{v} / \partial t + \rho(\vec{v} \cdot \vec{\nabla})\vec{v}$ , where the second term is called the "convective derivative." For steady flow we have  $\vec{f} = \rho(\vec{v} \cdot \vec{\nabla})\vec{v}$ . In these expressions  $\vec{v}$  is the average velocity of all the particles found in a fixed volume element of the fluid.

Note: for steady flow in a circular channel  $(\vec{v} \cdot \vec{\nabla})\vec{v} = -(v^2/r)\hat{r}$  in each volume element. This gives the net "centripetal acceleration" of the particles in the volume element. Since the flow is steady  $\partial \vec{v} / \partial t = 0$ .

*Bernoulli's Law*: For steady, incompressible and irrotational flow,  $(1/2)\rho v^2 + \rho g z + p = \text{constant}$ . This is the work energy theorem for steady flow in a fluid.